

Spectroscopy of a Reissner-Nordström black hole via an action variable

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Abstract With the help of the Bohr–Sommerfeld quantization rule, the area spectrum of a charged, spherically symmetric spacetime is obtained by studying an adiabatic invariant action variable. The period of the Einstein-Maxwell system, which is related to the surface gravity of a given spacetime, is determined by the Kruskal-like coordinates. It is shown that the area spectrum of the Reissner-Nordström black hole is evenly spaced and the spacing is the same as that of a Schwarzschild black hole, which indicates that the area spectrum of a black hole is independent of its parameters. In contrast to the quasi-normal mode analysis, we do not impose the small charge limit as the general area gap 8π is obtained.

Keywords Area spectrum · Action variable · Hamiltonian · Black hole

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1 Introduction

Investigation the properties of black holes has attracted attention of more and more astronomers and physicists in the past several years. On one hand, people want to uncover the mystery of black holes by detecting their fingerprints. On the other hand, black holes may be a base to test any scheme for a quantum theory of gravity. A crucial development in black hole physics is the discovery of Hawking [1] that a black hole was not totally black but can emit radiation with Hawking temperature $T_{BH} = \hbar \kappa_+ / 2\pi$, where κ_+ is the surface gravity at the event horizon. Based on the Hawking temperature, one can easily obtain the well known Bekenstein–Hawking entropy $S_{BH} = A/4l_p^2$, where A is the horizon area and $l_p = (G\hbar/c^3)^{1/2}$ is the Planck length.

The idea of quantization of a black hole was proposed first by Bekenstein [2–4], and was motivated by the analog that a black hole plays the role in gravitation as an atom plays in the quantum mechanics. He observed that the horizon area of a non-extremal black hole is classically adiabatic invariant. According to the Ehrenfest principle, any classical adiabatic invariant corresponds to a quantum entity with discrete spectrum; Bekenstein hence conjectured that the horizon area of a non-extremal quantum black hole has a discrete eigenvalue spectrum. Based on Christodoulou’s point particle model [5], Bekenstein found that the smallest possible increase in horizon area of a non-extremal black hole is $\Delta A = 8\pi l_p^2$, as the Heisenberg quantum uncertainty principle was considered [3]. Following the pioneering work of Bekenstein [3], a lot of independent calculations have been done and the uniformly spaced area spectrum was reproduced [6–10]. Later on, Hod [11] elucidated that the analysis of Bekenstein was only analogous to the well-known semiclassical determination of a lower bound on the ground state energy of the hydrogen atom, and that instead one should consider a wave analysis of black hole perturbations. In terms of Bohr’s correspondence principle, Hod found that the area spectrum of a black hole can be determined by the analysis of the asymptotic behavior (i.e., $n \rightarrow \infty$) of the highly damped quasi-normal mode frequencies. Using the real part of the quasi-normal mode frequency of the Schwarzschild black hole, the area spectrum was found to be $\Delta A = 4\lambda l_p^2$, in which $\lambda = \ln 3$. In 2002, Kunstatter [12] further confirmed this result, combining the proposal of Bekenstein that the black hole horizon area is adiabatic invariant and the proposal of Hod that the quasi-normal mode frequency is responsible for the area spectrum. According to Kunstatter’s viewpoint, the ratio M/ω is invariant, as the quasi-normal mode frequency ω and black hole mass M are treated as the classical vibrational frequency and system energy in large n limit. On the basis of statistical interpretation for black

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hole entropy, Bekenstein and Mukhanov [13] found that the parameter λ should be equal to $\ln k$, so the results of Hod and Kunstatter are believed to be more reasonable physically. In 2007, Maggiore [14] gave a new interpretation of the black hole quasi-normal mode frequencies. He found that the proper frequency of the equivalent harmonic oscillator, which is interpreted as the quasi-normal mode frequencies $\omega = \omega_R + i\omega_I$, should consider contributions of both the real part ω_R and imaginary part ω_I in high damping limit. More importantly, Maggiore found that the most interesting case is that for the highly excited quasi-normal modes, where the imaginary part rather than the real part is dominant. In this case, the area spectrum of a Schwarzschild black hole is found to be $\Delta A = 8\pi l_p^2$, which is consistent with the semiclassical result of Bekenstein. Based on the ideas of Hod, Kunstatter, and Maggiore, the area spectrum and entropy spectrum of many black hole spacetimes have been investigated [15]. It was shown that the spacings for the entropy spectrum and area spectrum are equidistant in Einstein gravity. In modified gravity theory, it was found that the entropy spectrum is equidistant, but the area spectrum is not equidistant [16]. As higher order quantum corrections to the semiclassical action are considered [17], the entropy spectrum and area spectrum obey the same rules as in modified gravity theory.

Very recently, Majhi and Vagenas [18] proposed another scheme to study black hole spectroscopy. In their work, they did not resort to the quasi-normal mode frequencies of a black hole to find the area spectrum. Instead, they used an adiabatic invariant $\int p_i dq_i$, which obeys the Bohr-Sommerfeld quantization rule; here p_i is the conjugate momentum of the coordinate q_i . Their idea was motivated mostly by the initial inference of Bekenstein that the horizon area of a black hole is adiabatic invariant. As the period of the gravity system is given, which was shown to be related to the surface gravity at the event horizon of the black hole, they obtained an equally spaced entropy spectrum of a static, spherical symmetry black hole with its quantum to be equal to the one given by Bekenstein.

In this paper, we intend to investigate the spectroscopy of a Reissner-Nordström (RN) black hole. For the area spectrum of a charged black hole, there have been many investigations. In terms of the reduced phase-space quantization, Barvinsky et al [19] found that the horizon area should be $A_{n,p} = 4\pi(2n + p + 1)l_p^2$, where $n, p = 0, 1, 2, \dots$ and the quantum number p corresponds to $Q = \pm\sqrt{\hbar p}$. Making use of Bohr's correspondence principle, Hod showed that the area spectrum was $\Delta A = 4\ln 2 l_p^2$ and $\Delta A = 4\ln 3 l_p^2$ respectively, for the event horizon area and the total areas of the inner horizon and outer horizon [20, 21]. Recently, Banerjee et al [22] got the value $\Delta A = 4l_p^2$ from the viewpoint of the tunneling paradigm. Based on the quasi-normal modes analysis, Wei et al [23] and Lopez-Ortega [24], respectively, ob-

tained the formalism $\Delta A = 8\pi l_p^2$ for the charged Garfinkle-Horowitz-Strominger black hole and D-dimensional RN black hole. Note that in Refs. [23, 24], the small charge limit was imposed. It is obvious that there is no unanimous consensus on the spacing of area spectrum of a charged black hole. According to the viewpoint of Bekenstein [3], the spacing is related to the total number of quantum states of the horizon; therefore an -n-depth understanding the spacing of area spectrum of a black hole may shed light on the statistics origin of black hole entropy or information.

Motivated by Ref. [18], we will investigate area spectrum of an RN black hole using an adiabatic invariant action variable. In analytical mechanics, it is known that the action I , action variable I_ν , and Hamiltonian H of any single periodic system satisfy the relation $I = I_\nu - \int H dt$. Thus as the action and Hamiltonian are given, the quantized action variable can be obtained with the help of the Bohr-Sommerfeld quantization rule. We will prove that the quantized action variable is nothing but the entropy of the black hole, and thus the entropy and further the horizon area of the black hole can be quantized. In the literature, it is indicated that the quantum effects of interest take place at the event horizon and that the physics near the horizon can be described effectively by a two-dimensional spacetime [25]. We hence restrict our investigation to the effective two-dimensional background.

The remainder of this paper is arranged as follows. In Sec.2, we will find the period of the Einstein-Maxwell system. In Sec.3, on the basis of the Bohr-Sommerfeld quantization rule, we will discuss the spacings of the entropy spectrum and area spectrum by studying the adiabatic invariant action variable. Sec.IV presents our conclusion.

2 Reissner-Nordstrom black hole in Kruskal-like coordinates

The line element of an RN black hole is

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2, \quad (1)$$

where

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} = \frac{(r - r_+)(r - r_-)}{r^2}, \quad (2)$$

in which $r_\pm = M \pm \sqrt{M^2 - Q^2}$ are the outer horizon and inner horizon and M, Q are the mass and charge of the black hole. The electromagnetic potential is

$$A_\mu = \left(\frac{Q}{r}, 0, 0, 0\right). \quad (3)$$

More and more literature publications have indicated that the interesting quantum phenomena, such as Hawking radiation, take place at the event horizon of a black hole, and near the horizon the higher dimensional background can effectively be reduced to a two dimensional spacetime [25]. In this paper, we are going to investigate the change in horizon

area of the RN black hole while a particle runs out from the event horizon, so we restrict our study to the effective two dimensional spacetime.

According to the dimensional reduction technique, the two-dimensional spacetime of an RN black hole can be written as [26]

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2. \quad (4)$$

In this spacetime, it is well known that the event horizon is a coordinate singularity. To get the period of the Einstein-Maxwell system, we should introduce the Kruskal-like coordinate. Without losing the integrity of this paper, we will give some key steps to construct the Kruskal-like coordinates.

As the first step of the coordinate transformations, we use the tortoise coordinate defined as

$$dr^* = \frac{1}{f(r)}dr. \quad (5)$$

Integrating it over r from 0 to r , we obtain

$$r^* = r + \frac{1}{2\kappa_+} \ln \frac{r-r_+}{r_+} + \frac{1}{2\kappa_-} \ln \frac{r-r_-}{r_-}, \quad (6)$$

in which $\kappa_{\pm} = \frac{r_+ - r_-}{2r_{\pm}^2}$ is the surface gravity on the outer (inner) horizon. Note that we have considered only the range $r > r_+$ here. Using the null coordinates $u = t - r^*$, $v = t + r^*$, we can construct the coordinates $U = -e^{-\kappa_+ u}$, $V = e^{\kappa_+ v}$. In this case, Eq. (4) can be rewritten as

$$ds^2 = -\kappa_+^{-2} e^{\kappa_+ (\mu - \nu)} f(r) dU dV. \quad (7)$$

Defining

$$T = \frac{1}{2}(V + U) = e^{\kappa_+ r} \left(\frac{r-r_+}{r_+} \right)^{\frac{1}{2}} \left(\frac{r-r_-}{r_-} \right)^{\frac{\kappa_+}{2\kappa_-}} \sinh \kappa_+ t, \quad (8)$$

$$R = \frac{1}{2}(V - U) = e^{\kappa_+ r} \left(\frac{r-r_+}{r_+} \right)^{\frac{1}{2}} \left(\frac{r-r_-}{r_-} \right)^{\frac{\kappa_+}{2\kappa_-}} \cosh \kappa_+ t, \quad (9)$$

Eq. (7) can be further expressed as

$$ds^2 = \kappa_+^{-2} e^{-2\kappa_+ r} \left(\frac{r-r_-}{r_+} \right) \left(\frac{r_-}{r-r_-} \right)^{\frac{\kappa_+}{\kappa_-}} (-dT^2 + dR^2), \quad (10)$$

in which T, R are the Kruskal coordinates. Extending the time coordinate to its imaginary axis, namely $t = -i\tau$, Eq. (8) and Eq. (9) can be rewritten as

$$iT = \frac{1}{2}(V + U) = e^{\kappa_+ r} \left(\frac{r-r_+}{r_+} \right)^{\frac{1}{2}} \left(\frac{r-r_-}{r_-} \right)^{\frac{\kappa_+}{2\kappa_-}} \sin \kappa_+ \tau, \quad (11)$$

$$R = \frac{1}{2}(V - U) = e^{\kappa_+ r} \left(\frac{r-r_+}{r_+} \right)^{\frac{1}{2}} \left(\frac{r-r_-}{r_-} \right)^{\frac{\kappa_+}{2\kappa_-}} \cos \kappa_+ \tau. \quad (12)$$

Obviously, both T, R are periodic functions with respect to the Euclidean time τ with period $2\pi/\kappa_+$. So the period for any continuous function on variable T, R is $2\pi/\kappa_+$ too. This period is very useful for studying Hawking temperature from the viewpoint of the temperature Green function [27]. Next, we will use this period to study the area spectrum of the RN black hole via an adiabatic invariant action variable.

3 Entropy spectrum and area spectrum

The adiabatic invariant is a notion in analytical mechanics. A physical system governed by a Hamiltonian $H(q, p, \lambda(t))$ is regarded as undergoing an adiabatic change if $\lambda(t)$ varies on a timescale longer than the longest timescale T of the internal motions, namely $T \frac{d\lambda}{dt} \ll \lambda$. According to the view of Ehrenfest, all Jacobi action integrals of the form $I_v = \oint p dq$ for a quasi-periodic system are adiabatic invariant action variable. In this section, we will employ the adiabatic invariant action variable I_v , which obeys the Bohr-Sommerfeld quantization rule $I_v = nh$, to investigate area spectrum of an RN black hole.

It is well known that in classical mechanics, the action I , action variable I_v , and the Hamiltonian H of any single periodic system satisfy the relation

$$I = I_v - \int H dt. \quad (13)$$

Obviously, to obtain the action variable, one should find the Hamiltonian of the system and the action of the moving particle. For any classical Einstein gravity system, the Hamiltonian (including its Gibbons-Hawking surface term) is always the Arnowitt-Deser-Misner (ADM) mass of the system. Based on Eq. (13), it was found that there is a ground state for the Schwarzschild black hole, which may be a candidate of dark matter [28].

For the charged background spacetime, the effect of the electromagnetic field on action variable must be considered. Firstly, we concentrate on the Hamiltonian. To investigate the first law of black hole mechanics, Wald [29, 30] once obtained the Hamiltonian for the Einstein-Maxwell system. It was found that the ADM Hamiltonian for Einstein-Maxwell theory has the form

$$H = \int_{\Sigma} (N^{\mu} C_{\mu} + N^{\mu} A_{\mu} C), \quad (14)$$

where Σ is a three-dimensional manifold, N^{μ} , A_0 are non-dynamical variables, and

$$0 = C = \frac{1}{4\pi} \sqrt{h} D_a E^a, \quad (15)$$

$$0 = C_0 = \frac{1}{16\pi} \sqrt{h} [-R + 2E_a E^a + F_{ab} F^{ab} + \frac{1}{h} (\pi_{ab} \pi^{ab} - \frac{1}{2} \pi^2)], \quad (16)$$

$$0 = C_a = -\frac{1}{8\pi} \sqrt{h} [D_b (\pi_a^b / \sqrt{h}) - 2F_{ab} E^b], \quad (17)$$

in which E^a is the electric field, π^{ab} is the momentum canonically conjugate to h_{ab} , and $F_{ab} = 2D_{[a} A_{b]}$ is the field strength. In the case where N^{μ} asymptotically approaches a time transition and considering contributions from boundary terms at infinity, the Hamiltonian can be written as

$$H = H_{\Sigma} + \frac{1}{16\pi} \oint_{\infty} dS^a [\partial^b h_{ab} - \partial_a h_a^b] + \frac{1}{4\pi} \oint_{\infty} dS^a A_0 E^a. \quad (18)$$

Taking into account the classical Hamiltonian constraints, H_Σ will vanish, and hence the Hamiltonian should be

$$H = \frac{1}{16\pi} \oint dS^a [\partial^b h_{ab} - \partial_a h_a^b] + \frac{1}{4\pi} \oint dS^a A_0 E^a = M + \phi^\infty Q, \quad (19)$$

in which $\phi = Q/r$ is the electrostatic potential and “ ∞ ” stands for the value at infinity. It should be pointed out that the gauge potential in Eq. (3) is divergent in the usual RN coordinates. So a transformation must be performed so that the vector potential will smooth through the horizon. In Ref. [31], the transformation $A'_a = A_a + \frac{Q}{r_+}(dt)_a$ has been used. Here we also adopt this skill. In this case, the Hamiltonian changes to

$$H = M - \phi^+ Q, \quad (20)$$

where $\phi^+ = Q/r_+$ is the electrostatic potential at the event horizon.

For the action, many works have been published [32]. It has been shown that the general coordinate $A_\mu = (A_t, 0)$ is an ignorable coordinate and the corresponding freedom should be eliminated completely. In this case, the action I can be written as

$$I = \int p_r dr - \int p_{A_t} dA_t, \quad (21)$$

where p_r , p_{A_t} are the canonical momenta conjugate to r and A_t , respectively. Therefore, for the two-dimensional Euclidean metric

$$ds^2 = f(r)d\tau^2 + f^{-1}(r)dr^2, \quad (22)$$

the invariant action variable can be expressed as

$$\begin{aligned} \int H d\tau + \int p_r dr - \int p_{A_t} dA_t \\ = \int H d\tau + \int \int_0^{p_r} dp'_r dr - \int \int_0^{p_{A_t}} dp'_{A_t} dA_t, \end{aligned} \quad (23)$$

in which $\tau = -it$ is the Euclidean time. Considering the following Hamilton's equations:

$$\dot{r} = \frac{dH}{dp'_r} \big|_{(r; A_\tau, p_{A_\tau})}, \quad (24)$$

$$\dot{A}_\tau = \frac{dH}{dp'_{A_\tau}} \big|_{(A_\tau; r, p_{A_\tau})}, \quad (25)$$

the invariant action variable can be rewritten as

$$\begin{aligned} \oint pdq = \int H d\tau + \int \int_0^{H(r; A_\tau, p_{A_\tau})} \frac{dH'_{(r; A_\tau, p_{A_\tau})}}{\dot{r}} dr \\ - \int \int_0^{H(A_\tau; r, p_{A_\tau})} \frac{dH'_{(A_\tau; r, p_{A_\tau})}}{\dot{r}} dr. \end{aligned} \quad (26)$$

To obtain \dot{r} , one should consider the radial, null geodesics while a particle runs out. This method has been used to study the tunneling effect extensively [32]. Solving Eq. (22), we get

$$\dot{r} \equiv \frac{dr}{d\tau} = \pm if(r), \quad (27)$$

where \pm denotes the outgoing (incoming) radial null paths. Thereafter, we only focus on the outgoing paths, since these are the ones related to the quantum mechanically nontrivial features under consideration. In this case, Eq. (26) change to

$$\oint pdq = \int H d\tau + \int \int_0^M dM' d\tau - \int \int_0^Q \phi dQ' d\tau, \quad (28)$$

where we have used

$$dH'_{(r; A_\tau, p_{A_\tau})} = dM', \quad (29)$$

$$dH'_{(A_\tau; r, p_{A_\tau})} = \phi dQ'. \quad (30)$$

Incorporating Eq. (20), Eq. (28) can be further simplified as

$$\oint pdq = 2 \int \int_{(0,0)}^{(M,Q)} (dM' - \frac{Q}{r_+} dQ') d\tau. \quad (31)$$

Since we are considering only the outgoing paths, the integration limit for τ should be $[0, \frac{\pi}{\kappa_+}]$ [18]. Using the relation

$$T_{BH} = \frac{\hbar \kappa_+}{2\pi}, \text{ we can obtain}$$

$$\oint pdq = \hbar \int_{(0,0)}^{(M,Q)} \frac{1}{T_{BH}} (dM' - \frac{Q}{r_+} dQ'). \quad (32)$$

Simplifying Eq. (32), we get

$$\oint pdq = \hbar S_{BH}, \quad (33)$$

where the first law of black hole thermodynamics, $dM = T_{BH} dS + \phi_+ dQ$, has been used. Implementing the Bohr-Sommerfeld quantization rule

$$\oint pdq = 2\pi n \hbar, \quad (34)$$

the black hole entropy spectrum can be given directly as

$$S_{BH} = 2\pi n, n = 1, 2, 3, \dots \quad (35)$$

One can also get the spacing for the entropy spectrum

$$\Delta S_{BH} = 2\pi(n+1) - 2\pi n = 2\pi. \quad (36)$$

Obviously the entropy of an RN black hole is discrete and the spacing is equidistant. Recalling that the black hole entropy is proportional to its horizon area

$$S_{BH} = \frac{A}{4l_p^2}, \quad (37)$$

we can also get the area spectrum

$$\Delta A = 8\pi l_p^2. \quad (38)$$

It is evident that the area spectrum of the RN black hole is also equidistant. Our result confirms the initial proposal of Bekenstein further that the area spectrum is independent of the black hole parameters and the spacing is $8\pi l_p^2$.

For the spacing of the area spectrum of a black hole, Medved [33] argued that 8π is, by far, the most qualified candidate on the basis of the recent tunneling framework in Ref. [34]. His argument is supported by the emergent gravity proposed by Verlinde [35]. To obtain Newton's second law of mechanics and Newton's law of gravitation from thermodynamics, one finds that the unique choice of the minimal change in the entropy is $\Delta S = 2\pi$, namely, the area gap should be 8π . Our result, obviously, agrees with this argument.

4 Conclusions

The area spectrum of an RN black hole was investigated with the help of the Bohr-Sommerfeld quantization rule. We obtained an equally spaced area spectrum by studying the action variable of the Einstein-Maxwell system. Our result is the same as the one for a Schwarzschild black hole [18], which confirms the proposal of Bekenstein that the area spectrum of a black hole is independent of its parameters. In our calculation, the quasi-normal mode frequency is not used, so there is no confusion as to whether the real part or imaginary part is responsible for the area spectrum. This is more convenient and simple. More importantly, the small charge limit for a Reissner-Nordström black hole was not imposed, which is necessary from the viewpoint of quasi-normal mode analysis. Also, we should point out the difference of this paper compared with the work in Ref. [18], where the area spectrum of black holes was investigated using an adiabatic invariant. The relation between the action variable and adiabatic invariant has been discussed recently in Ref.[36]. It was found that only the action variable can be quantized as the equally spaced form $I_\nu = 2\pi n\hbar$, though the adiabatic invariant can also be quantized. For example, if the action variable I_ν can be quantized via Bohr-Sommerfeld quantization, I_ν^2 is not equally spaced even though it is an adiabatic invariant. Hence we think the action variable used in this paper is more reasonable

Our study is also valid for the rotating black hole. For a Kerr black hole, there is an ergosphere between the outer horizon and infinite redshift surface, so matter dropping into this region will be dragged inevitably. To avoid the dragging effect, one should often perform a dragging coordinate transformation. Recently, it was shown that the ergosphere outside a Kerr black hole is similar to the electromagnetic field outside an RN black hole in the reduced two-dimensional metric. The evidence stems from the investigation of Iso et al [37] that the partial wave of quantum fields in the four-dimensional rotating black hole background can be interpreted as a (1+1)-dimensional charged field with a charge proportional to the azimuthal angular momentum m . Thus the area spectrum of a Kerr black hole can be investigated by the method used for an RN black hole.

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